1 Integrieren und Differenzieren

1.1 Differenzialrechnung

$$(u+v)' = u' + v'$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v + uv'}{v^2}$$

$$\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$$

$$(cf)' = cf'$$

$$\left(f^{-1}\right)' = \frac{1}{f'\left(f^{-1}\right)}$$

1.2 Integralrechnung

$$\int u(x)\, v'(x)\, dx = u(x)\, v(x) - \int u'(x)\, v(x)\, dx + C$$

$$\int f(y)\, dy = \int f(g(x))\, g'(x)\, dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

1.3 Funktionen

Funktion → Ableitungsfunktion → Stammfunktion

$$k \to 0 \to kx + C$$

$$x^a \to ax^{a-1} \to \frac{x^{a+1}}{a+1}$$

$$\frac{1}{x} \to -\frac{1}{x^2} \to \ln|x|$$

$$e^x \to e^x \to e^x$$

$$a^x \to a^x \ln(a) \to \frac{a^x}{\ln(a)}$$

$$\ln(x) \to \frac{1}{x} \to x\ln(x) - x$$

$${}^a\log(x) \to \frac{1}{x}\frac{1}{\ln(a)} \to \frac{1}{\ln(a)}(x\ln(x) - x)$$

$$\sin \to \cos \to -\cos$$

$$\cos \to -\sin \to \sin$$

$$\tan \to \frac{1}{\cos^2} \to -\ln|\cos|$$

$$\sin(x\,y) \to y\,\cos(x\,y) \to -\frac{\cos(x\,y)}{y}$$

$$\arcsin \to \frac{1}{\sqrt{1-x^2}} \to x\arcsin x + \sqrt{1-x^2}$$

$$\arccos \to -\frac{1}{\sqrt{1-x^2}} \to x\arccos x - \sqrt{1-x^2}$$

$$\arctan \to \frac{1}{x^2+1} \to x\,\arctan x - \frac{\log(x^2+1)}{2}$$

$$\sinh \to \cosh \to \cosh$$

$$\cosh \to \sinh \to \sinh$$

$$\tanh \to \operatorname{sech}^2 \to \ln\cosh$$

$$\operatorname{asinh} \to \frac{1}{\sqrt{x^2+1}} \to x\,\operatorname{asinh} x - \sqrt{x^2+1}$$

$$\operatorname{acosh} \to \frac{1}{\sqrt{x^2-1}} \to x\,\operatorname{acosh} x - \sqrt{x^2-1}$$

$$\operatorname{atanh} \to \frac{1}{1-x^2} \to \frac{\log(1-x^2)}{2} + x\operatorname{atanh} x$$